International Journal of Solids and Structures 41 (2004) 3491–3504

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Chaos attitude motion and chaos control in an automotive wiper system

Shun-Chang Chang *, Hai-Ping Lin

Department of Mechanical and Automation Engineering, Da-Yeh University, 112 Shan-Jiau Road, Da-Tsuen, Chang-Hua, Taiwan 51505, R.O.C.

Received 19 August 2003; received in revised form 26 January 2004
Available online 11 March 2004

Abstract

The objective of this paper is to analyze chaotic motion and its control in an automotive wiper system, which consists of two blades driven by a DC motor via one link. The dynamical behaviors are numerically investigated by means of time responses, Poincare maps and frequency spectra. By using largest Lyapunov exponents, the periodic and chaotic motions are verified. Finally, the state feedback control method is applied to control chaotic motions effectively.

Keywords: Wiper system; Largest Lyapunov exponent; State feedback control; Chaotic motion

1. Introduction

Running a wiper system on a car windshield leads to many vibratory phenomena that may be harmful to the driver. Chatter vibrations are often observed in an automotive wiper. Their occurrence spoils comfortable driving. To find an effective way to control chatter vibrations, we attempt to clarify the behavior of the wiper system. The analysis of chatter vibrations performed by Suzuki and Yasuda (1998), leads to the following conclusions: Firstly, chattering is a self-excited vibration based on a stick-slip phenomenon. Secondly, the blade only vibrates within a certain range of speed. Beyond this range, the chatter vibrations no longer occur. This second property is a special feature of the stick-slip phenomenon as can be observed in other physical systems (Tarng and Cheng, 1995; Mokhtar et al., 1998; Oancea and Laursen, 1998).

A number of numerical analyses, such as bifurcation diagram, phase portraits, Poincare map, frequency spectra and Lyapunov exponents, are used to study the dynamical behaviors of the wiper system. For a broad range of parameters, the Lyapunov exponent is the most powerful method to measure the sensitivity of the dynamical system to change in initial conditions. It can help us to examine whether the system is in chaotic motion or not. The algorithms for computing Lyapunov exponents of smooth dynamical systems

* Corresponding author. Tel.: +886-4851-1888; fax: +886-4851-1224.
E-mail address: changsc@mail.dyu.edu.tw (S.-C. Chang).

0020-7683/$ - see front matter © 2004 Published by Elsevier Ltd.
doi:10.1016/j.ijsolstr.2004.02.005

are well developed (Shimada and Nagashima, 1979; Wolf et al., 1985; Benettin et al., 1980a,b). But there are nonsmooth dynamical systems with discontinuities, where this algorithm cannot be directly applied, for example, in machine dynamics due to dry friction, backlash, or impact. However, the methods of the calculation of Lyapunov exponents for nonsmooth dynamical systems have been proposed only in several papers (Muller, 1995; Hinrichs et al., 1997; Stefanski, 2000). The estimated method of the largest Lyapunov exponent for wiper system proposed by Stefanski (2000) is used in this paper.

Although chaotic behavior may be acceptable, it is, on the whole, undesirable since it can will degrade performance and restrict the operating range of many electric and mechanic devices. Recently, the control of chaotic stick-slip mechanical system is being further developed, several techniques have been proposed in Galvanetto (2001), Dupont (1991), and Feeny and Moon (2000). Galvanetto (2001) applied the adaptive control to control unstable periodic orbits embedded in chaotic attractors of some discontinuous mechanical systems. Feeny and Moon (2000) have applied high-frequency excitation, or dither, to quench stick-slip chaos. In order to improve the wiper system performance or avoid chatter vibration in an automotive wiper, we have to control a chaotic motion to a periodic orbit to a steady state. For this purpose, a simple control method proposed by Cai et al. (2002) is used in this paper to convert chaos into periodic motion through adapting linear state feedback of an available system variable.

2. Formulation of problem

A front wiper system has two blades. They are attached to the windshield at the driver's side and the passenger's side. Each blade is supported by an arm, which moves to and fro around the pivot. This motion is given by the rotating motion of a DC motor via a pantographic link. The schematic diagram of automotive wiper system is shown in Fig. 1. In this figure, the symbols with subscripts D and P are referred to as driver's and passenger's side, respectively. The lines L_i represent no deflection positions. The symbols θ_i $(i = \mathrm{D}, \mathrm{P})$ are the angular deflections with respect to the position L_i while the notations $\dot{\psi}_i$ are the angular velocity of the arms. The symbols l_i represent the length of the wiper arm and $\dot{z}_i$ represent relative velocities of the blades with respect to L_i at the position of the top of the wiper arms. Then:

$$\dot{z}_i = (\dot{\theta}_i + \dot{\psi}_i)l_i \quad (i = \mathrm{D}, \mathrm{P}). \tag{1}$$

In accordance with Newton's second law, the governing equations for a wiper on the i's $(i = \mathrm{D}, \mathrm{P})$ side can be expressed as follows (Suzuki and Yasuda, 1998):

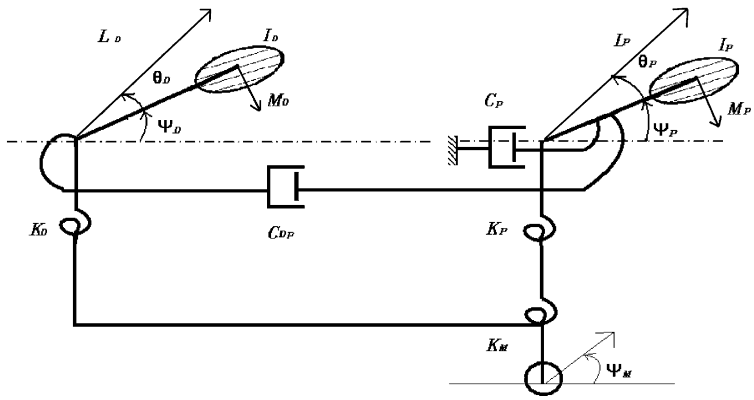

Fig. 1. The wiper system.

$$
\begin{aligned}
&\text{when}\\
&\quad \dot{z}_i \neq 0,\\
&\quad I_i\ddot{\theta}_i = -R_i - D_i - M_i(\dot{z}_i),\\
&\text{when}\\
&\quad \dot{z}_i = 0, |R_i| \geqslant N_i l_i \mu_0,\\
&\quad I_i\ddot{\theta}_i = -R_i - D_i - M_i(\dot{z}_i),\\
&\text{when}\\
&\quad \dot{z}_i = 0, |R_i| < N_i l_i \mu_0,\\
&\quad I_i\ddot{\theta}_i = 0(\dot{\theta}_i = -\dot{\psi}_i),
\end{aligned}
\tag{2}
$$

where the symbols I_i are the moments of inertia and M_i are the moments induced by the friction force between the wiper blades and the windshield. R_i and D_i are the moments produced by the restoring force and the damping force, respectively. That is given as follows:

$$R_{\mathrm{D}} = k_{\mathrm{D}}\theta_{\mathrm{D}} - k_{\mathrm{DP}}\theta_{\mathrm{P}}, \quad R_{\mathrm{P}} = k_{\mathrm{P}}\theta_{\mathrm{P}} - k_{\mathrm{PD}}\theta_{\mathrm{D}}, \tag{3}$$

$$D_{\mathrm{D}} = c_{\mathrm{D}}\dot{\theta}_{\mathrm{D}} - c_{\mathrm{DP}}\dot{\theta}_{\mathrm{P}}, \quad D_{\mathrm{P}} = c_{\mathrm{P}}\dot{\theta}_{\mathrm{P}} - c_{\mathrm{PD}}\dot{\theta}_{\mathrm{D}}, \tag{4}$$

where

$$
\begin{aligned}
&k_{\mathrm{D}} = K_{\mathrm{D}}(K_{\mathrm{P}} + K_{\mathrm{M}})/(K_{\mathrm{D}} + K_{\mathrm{P}} + K_{\mathrm{M}}), \quad k_{\mathrm{DP}} = k_{\mathrm{PD}} = K_{\mathrm{D}}K_{\mathrm{P}}/(K_{\mathrm{D}} + K_{\mathrm{P}} + K_{\mathrm{M}}),\\
&k_{\mathrm{P}} = K_{\mathrm{P}}(K_{\mathrm{D}} + K_{\mathrm{M}})/(K_{\mathrm{D}} + K_{\mathrm{P}} + K_{\mathrm{M}}), \quad c_{\mathrm{D}} = C_{\mathrm{DP}}, \quad c_{\mathrm{P}} = C_{\mathrm{DP}} + C_{\mathrm{P}},\\
&c_{\mathrm{DP}} = c_{\mathrm{PD}} = C_{\mathrm{DP}}.
\end{aligned}
$$

The moments M_i can be written as:

$$M_i(\dot{z}_i) = N_i l_i \mu(\dot{z}_i), \tag{5}$$

where N_i is the normal force. The coefficient of friction, μ, which can be expressed using the following relationship proposed by (Suzuki and Yasuda, 1995):

Table 1
Parameter values of the wiper system

System parameter	Value	Unit
I_{D}	4.07×10^{-2}	kg m^2
I_{P}	3.67×10^{-2}	kg m^2
K_{D}	7.20×10^{-2}	N m/rad
K_{P}	7.51×10^{-2}	N m/rad
K_M	3.53×10^{-2}	N m/rad
C_{DP}	1.00×10^{-2}	N m s/rad
C_{P}	1.00×10^{-2}	N m s/rad
l_{P}	4.50×10^{-1}	m
l_{D}	4.70×10^{-1}	m
ψ_{P}	$1.16\times\psi_{\mathrm{D}}$	rad/s
N_{D}	7.35	N
N_{P}	5.98	N
μ_{D}	1.18	
μ_1	-9.84×10^{-2}	
μ_2	4.74×10^{-1}	

$$\mu(\dot{z}_i) = \mu_0 \ \text{sgn}(\dot{z}_i) + \mu_1 \dot{z}_i + \mu_2 \dot{z}_i^3 \quad (i = \text{D}, \text{P}). \tag{6}$$

Let $x_1 = \theta_D, x_2 = \dot{\theta}_D, x_3 = \theta_P$ and $x_4 = \dot{\theta}_P$ be the state variables, the state equations of the wiper system (Eq. (2)) on the driver's side can be written as follows:

$$\begin{aligned}
&\text{when}\\
&\quad \dot{z}_D \neq 0,\\
&\quad \dot{x}_1 = x_2,\\
&\quad \dot{x}_2 = (-R_D - D_D - M_D(\dot{z}_D))/I_D,\\
&\text{when}\\
&\quad \dot{z}_D = 0, |R_D| \geqslant N_D l_D \mu_0,\\
&\quad \dot{x}_1 = x_2,\\
&\quad \dot{x}_2 = (-R_D - D_D - M_D(\dot{z}_D))/I_D,\\
&\text{when}\\
&\quad \dot{z}_D = 0, |R_D| < N_D l_D \mu_0,\\
&\quad x_2 = -\dot{\psi}_D,\\
&\quad \dot{x}_1 = x_2,\\
&\quad \dot{x}_2 = 0.
\end{aligned} \tag{7a}$$

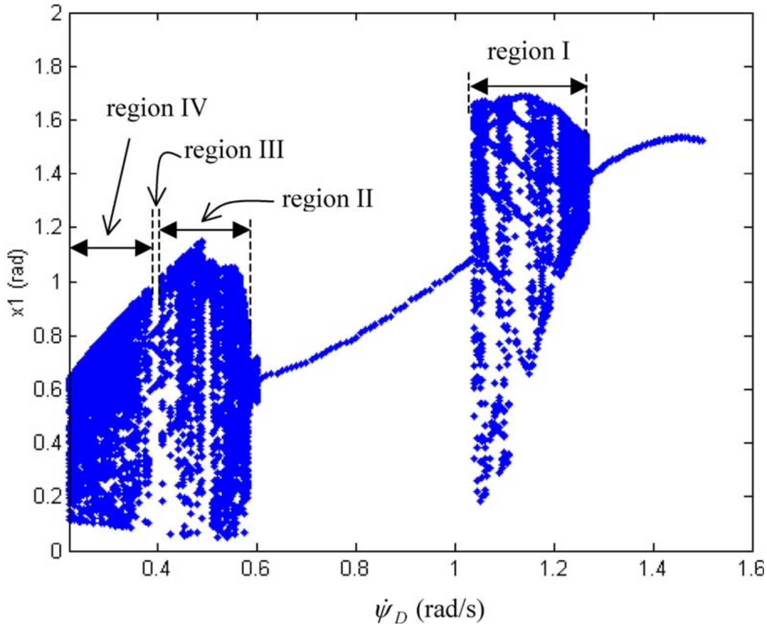

Fig. 2. Bifurcation diagram of the angular velocity of the arm of driver's side $\dot{\psi}_D$ versus angular deflection x_1.

The state equations of the wiper system (Eq. (2)) on the passenger's side can be written as follows:

$$
\begin{aligned}
&\text{when} \\
&\quad \dot{z}_{\mathrm{P}} \neq 0, \\
&\quad \dot{x}_3 = x_4, \\
&\quad \dot{x}_4 = (-R_{\mathrm{P}} - D_{\mathrm{P}} - M_{\mathrm{P}}(\dot{z}_{\mathrm{P}}))/I_{\mathrm{P}}, \\
&\text{when} \\
&\quad \dot{z}_{\mathrm{P}} = 0, |R_{\mathrm{P}}| \geqslant N_{\mathrm{P}} l_{\mathrm{P}} \mu_0, \\
&\quad \dot{x}_3 = x_4, \\
&\quad \dot{x}_4 = (-R_{\mathrm{P}} - D_{\mathrm{P}} - M_{\mathrm{P}}(\dot{z}_{\mathrm{P}}))/I_{\mathrm{P}}, \\
&\text{when} \\
&\quad \dot{z}_{\mathrm{P}} = 0, |R_{\mathrm{P}}| < N_{\mathrm{P}} l_{\mathrm{P}} \mu_0, \\
&\quad x_4 = -\dot{\psi}_{\mathrm{P}}, \\
&\quad \dot{x}_3 = x_4, \\
&\quad \dot{x}_4 = 0.
\end{aligned}
\tag{7b}
$$

These values of the parameters of the above equations are listed in Table 1.

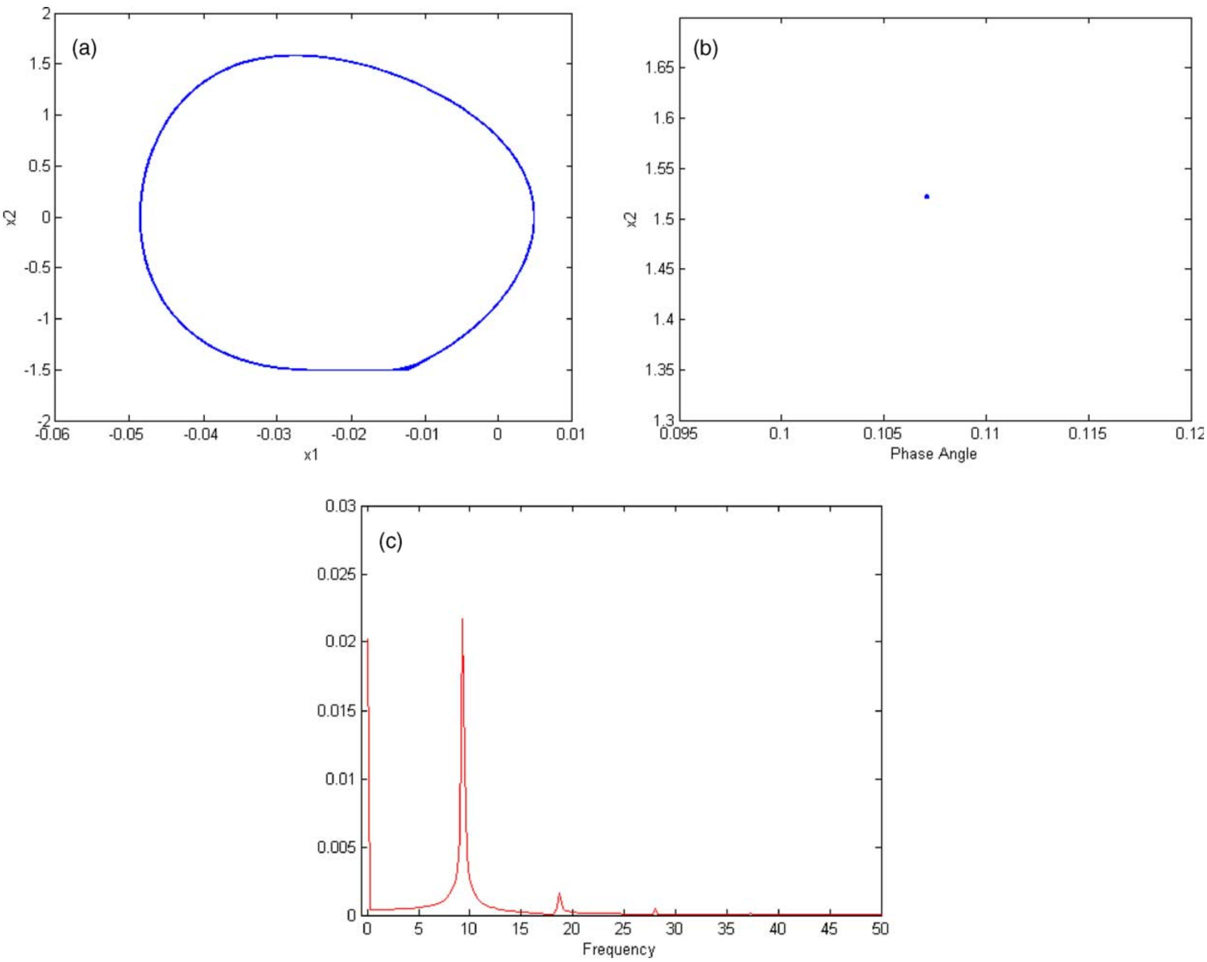

Fig. 3. Period-one orbit for $\dot{\psi}_{\mathrm{D}} = 1.5$ (rad/s): (a) phase portrait; (b) Poincare map; and (c) frequency spectrum.

3. The overall characteristics of the system and chaos attitude motion

To clearly understand the characteristics of this system, we carry out a series of numerical simulations from Eqs. (7). The resulting bifurcation diagram is shown in Fig. 2. The dynamic behavior may be observed more completely over a range of parameter values by the bifurcation diagram. It is a widely used technique to describe a transition from periodic motion to chaotic motion for a dynamical system. It can be clearly seen from this figure that the chaotic motions appear approximately at regions II and IV. Period-three motion appears in region III and period-n orbits display in region I. Here, each type of response is characterized by a phase portrait, Poincare map (velocity vs. phase angle), and frequency spectrum.

Fig. 3 shows the period-one solution. In other words, while wiping speed is high enough, equilibrium point of Eqs. (7) is stable. This means that no chatter vibrations will occur. This stable situation continues until the wiper speed decreases into the region I in Fig. 2, the stable period-n orbits, such as period-five orbit (as shown in Fig. 4) and period-seven orbit (see Fig. 5) or a quasi-periodic motion (presented in Fig. 6), namely "torus motion" that produces by two incommensurate frequencies, appear to this system. As the wiper speed continues to decrease into the regions II and IV in Fig. 2, the chaotic vibrations take place. This means that the chatter vibration occurs. The particular features of two descriptors characterize the essence

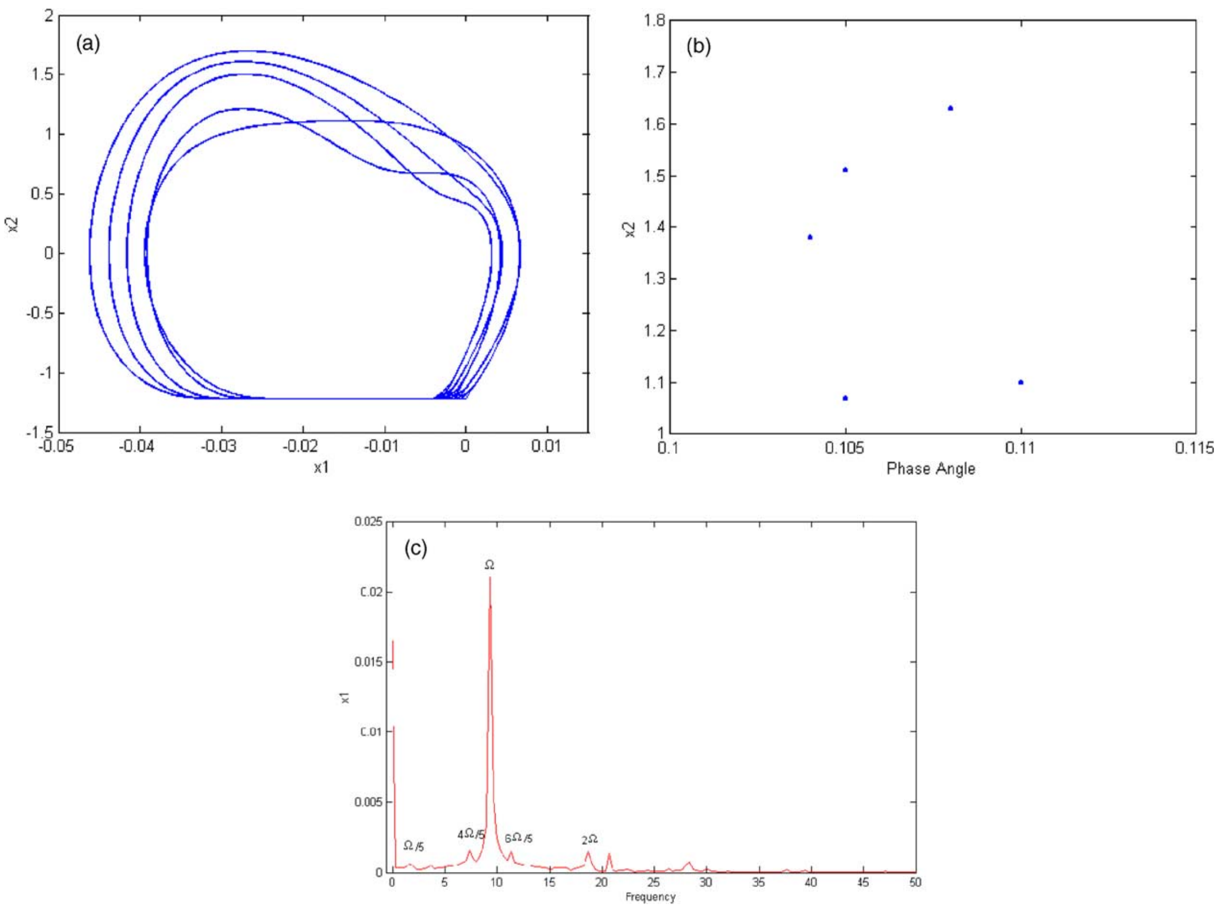

Fig. 4. Period-five orbit for $\dot{\psi}_D = 1.215$ (rad/s): (a) phase portrait; (b) Poincare map; and (c) frequency spectrum.

of the chaotic behavior: the Poincare map and the frequency spectrum. The Poincare map shows an infinite set of points referred to as a strange attractor. Simultaneously, the frequency spectrum of chaotic motion contains a broad band. The two features that strange attractor and continuous type Fourier spectrum are strong indicators of chaos. Their phase portraits, Poincare maps, and frequency spectra are shown in Figs. 7 and 8, respectively. The period-three bifurcation occurs in region III in Fig. 2, which eventually results in a chaotic motion. To see this behavior in detail, phase portrait, Poincare map, and frequency spectrum are shown in Fig. 9.

4. Chaotic behavior

In this section, we wish to demonstrate that the automotive wiper system has chaotic behavior, by computing the maximal Lyapunov exponent. Any system containing at least one positive Lyapunov exponent is defined to be chaotic. Lyapunov exponents are a measure of the rate of divergence (or convergence) of two initial nearby orbits. Recently, Stefanski (2000) has suggested a simple and effective method of estimation of the largest Lyapunov exponent, which utilizes the properties of synchronization

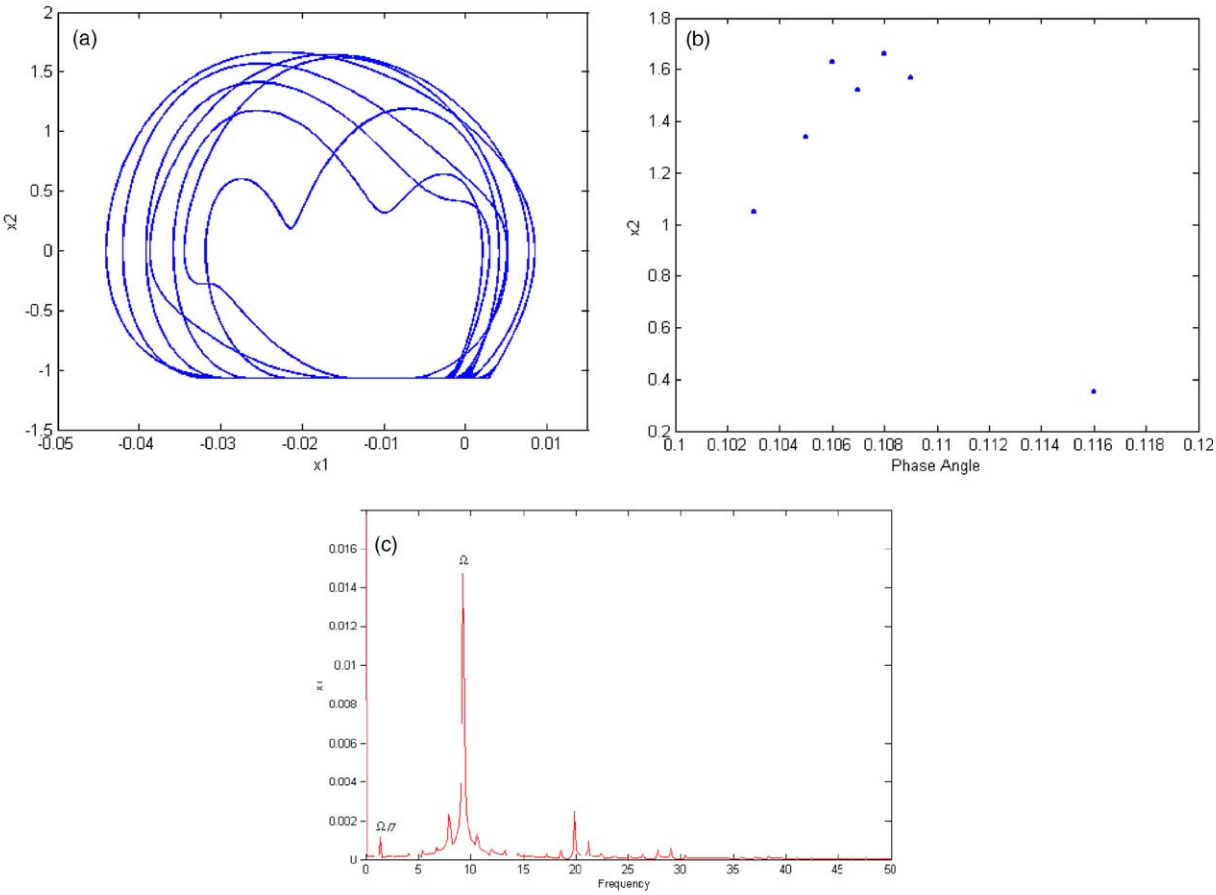

Fig. 5. Period-seven orbit for $\dot{\psi}_D = 1.068$ (rad/s): (a) phase portrait; (b) Poincare map; and (c) frequency spectrum.

phenomenon. This method can be explained briefly: the dynamical system is decomposed into two subsystems as follows:

drive system:

$$\dot{x} = f(x), \tag{8}$$

response system:

$$\dot{y} = f(y). \tag{9}$$

Consider a dynamical system, which is composed, of two identical n-dimensional subsystems, where only the response system (8) is combined with a coupling coefficient d, while the equation of drive remain the same. The first order differential equations describing such a system can be written as:

$$\dot{x} = f(x), \quad \dot{y} = f(y) + d. \tag{10}$$

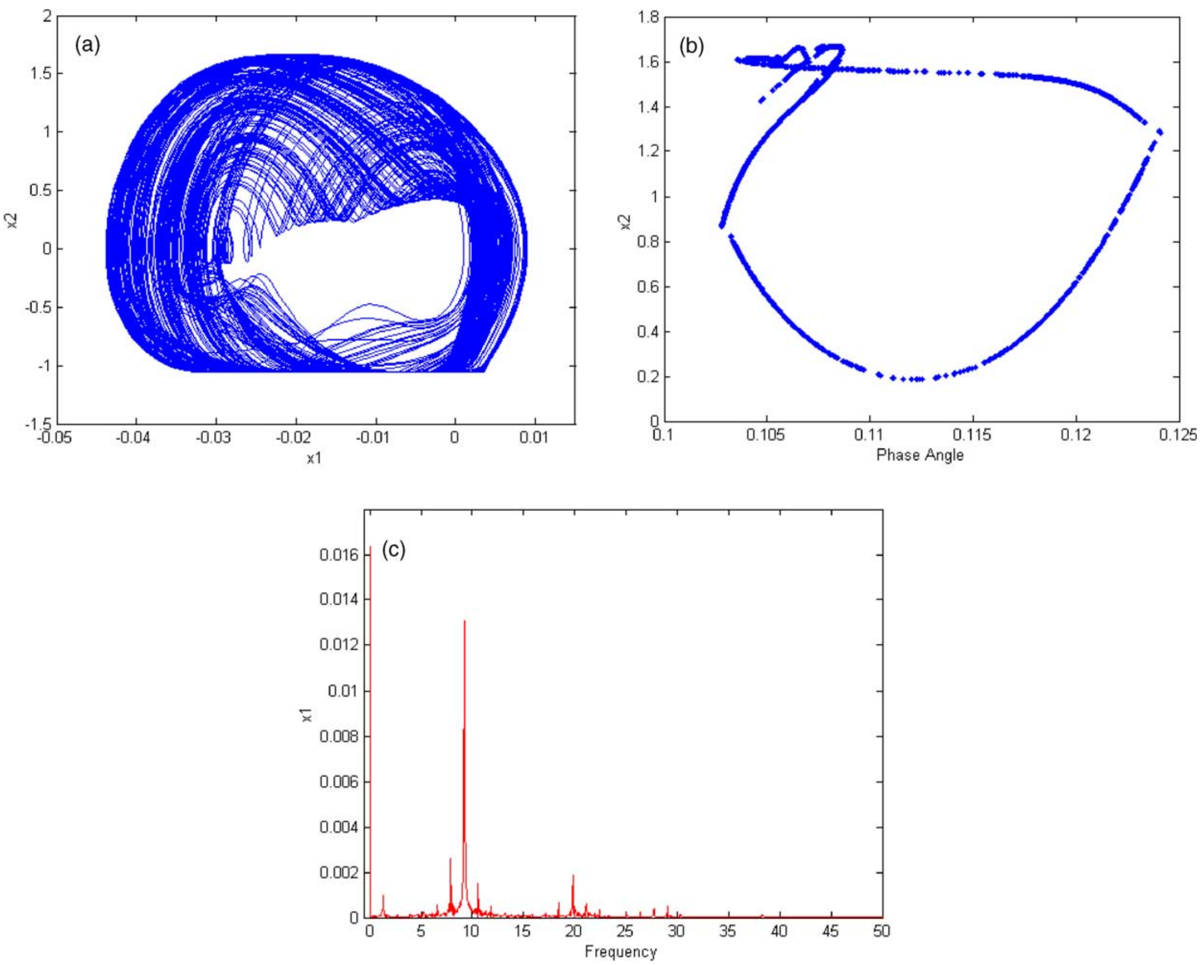

Fig. 6. Quasi-periodic motion for $\dot{\psi}_D = 1.054$ (rad/s): (a) phase portrait; (b) Poincare map; and (c) frequency spectrum.

Now the condition of synchronization (Eq. (10)) is given by the inequality:

$$d > \lambda_{\max}. \tag{11}$$

The smallest value of the coupling coefficient d, for which the synchronization takes place d_s is assumed to be equal to the maximum Lyapunov exponent:

$$d_s = \lambda_{\max}. \tag{12}$$

The results of numerical calculations are listed in Fig. 10, which shows the largest Lyapunov exponents that have been obtained using the described synchronization method.

5. Controlling chaos

In order to improve the performance of a dynamic system or avoid the chaotic behaviors, we need to control a chaotic system to a periodic motion, which is beneficial for working with a particular condition. It is thus of great practical importance to develop suitable control methods. Recently, Cai et al. (2002) have

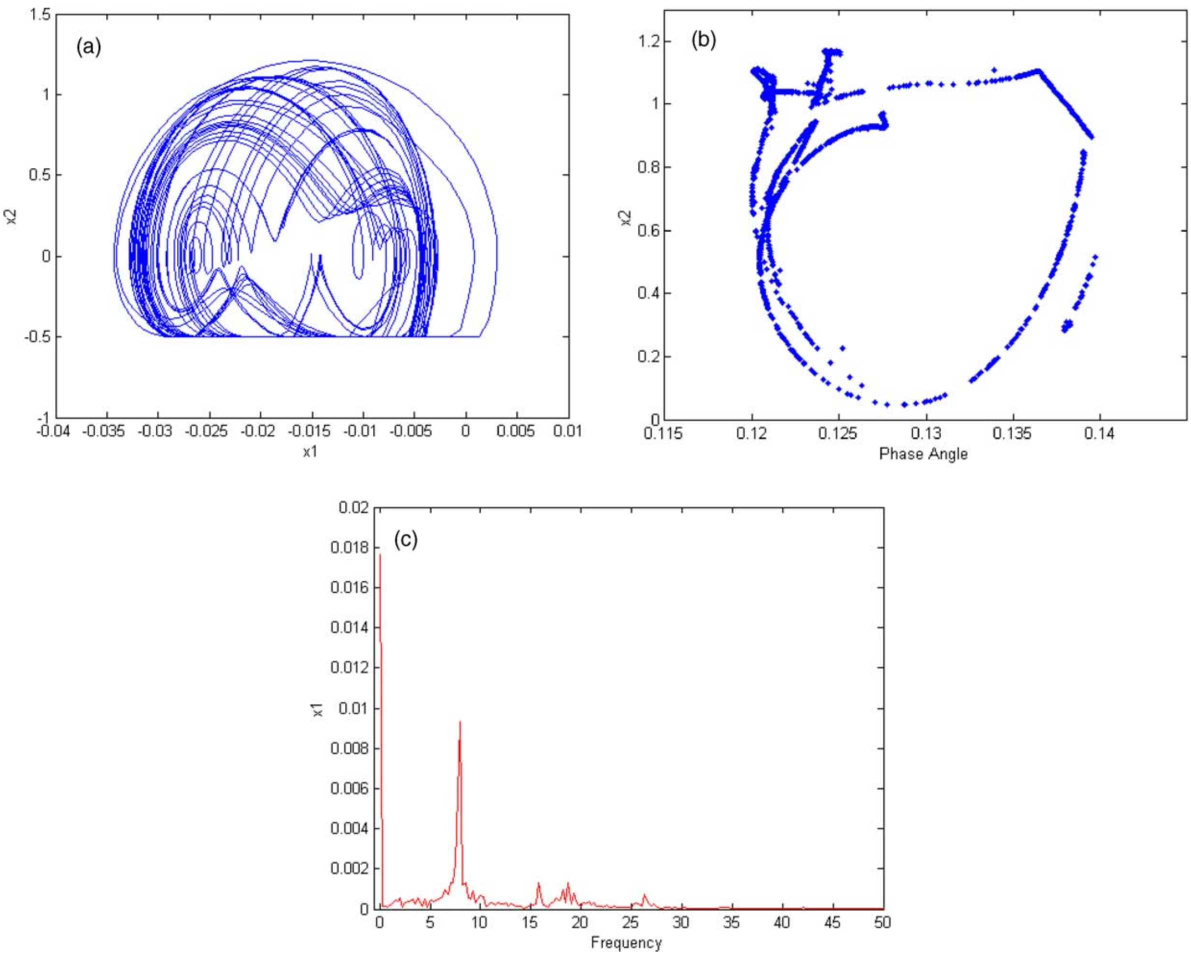

Fig. 7. Chaotic motion for $\dot{\psi}_D = 0.5$ (rad/s): (a) phase portrait; (b) Poincare map; and (c) frequency spectrum.

suggested a simple and effective control method, which converts chaos into periodic motion by using linear state feedback of an available system variable. This method can be explained briefly: the n-dimensional dynamical system

$$\dot{x} = f(x, t), \tag{13}$$

where $x(t) \in R^n$ is the state vector, and $f = (f_1, \ldots, f_i, \ldots, f_n)$, where f_i is linear or nonlinear function and f includes at least one nonlinear function. Suppose $f_k(x, t)$ is the nonlinear function that leads to chaotic motion in system (13), then only one term of state feedback of available system variable x_m is added to the equation that includes f_k as follows:

$$\dot{x}_k = f_k(x, t) + Gx_m, \quad k, m \in \{1, 2, \ldots, n\}, \tag{14}$$

where G is feedback gain, and other functions keep their original forms.

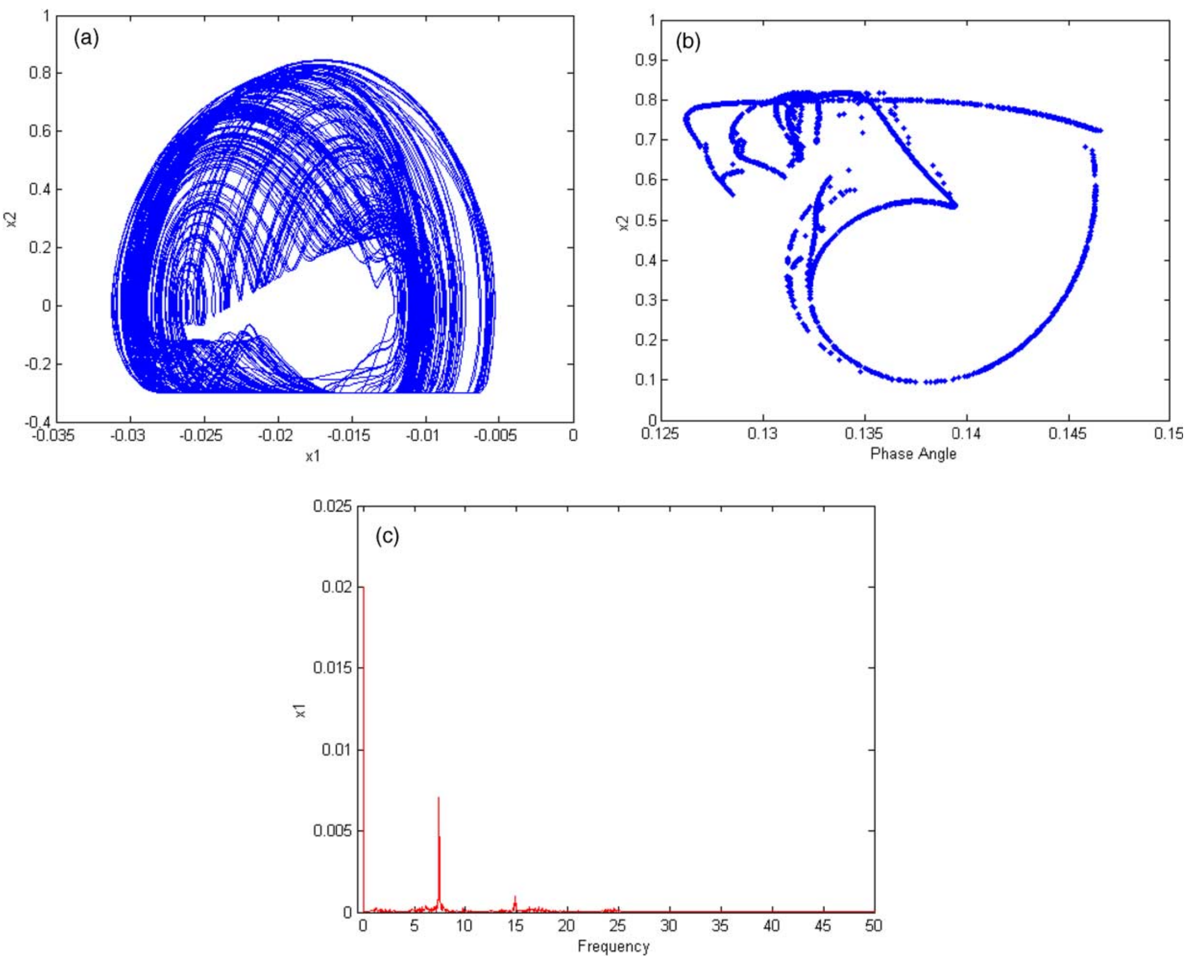

Fig. 8. Chaotic motion for $\dot{\psi}_D = 0.3$ (rad/s): (a) phase portrait; (b) Poincare map; and (c) frequency spectrum.

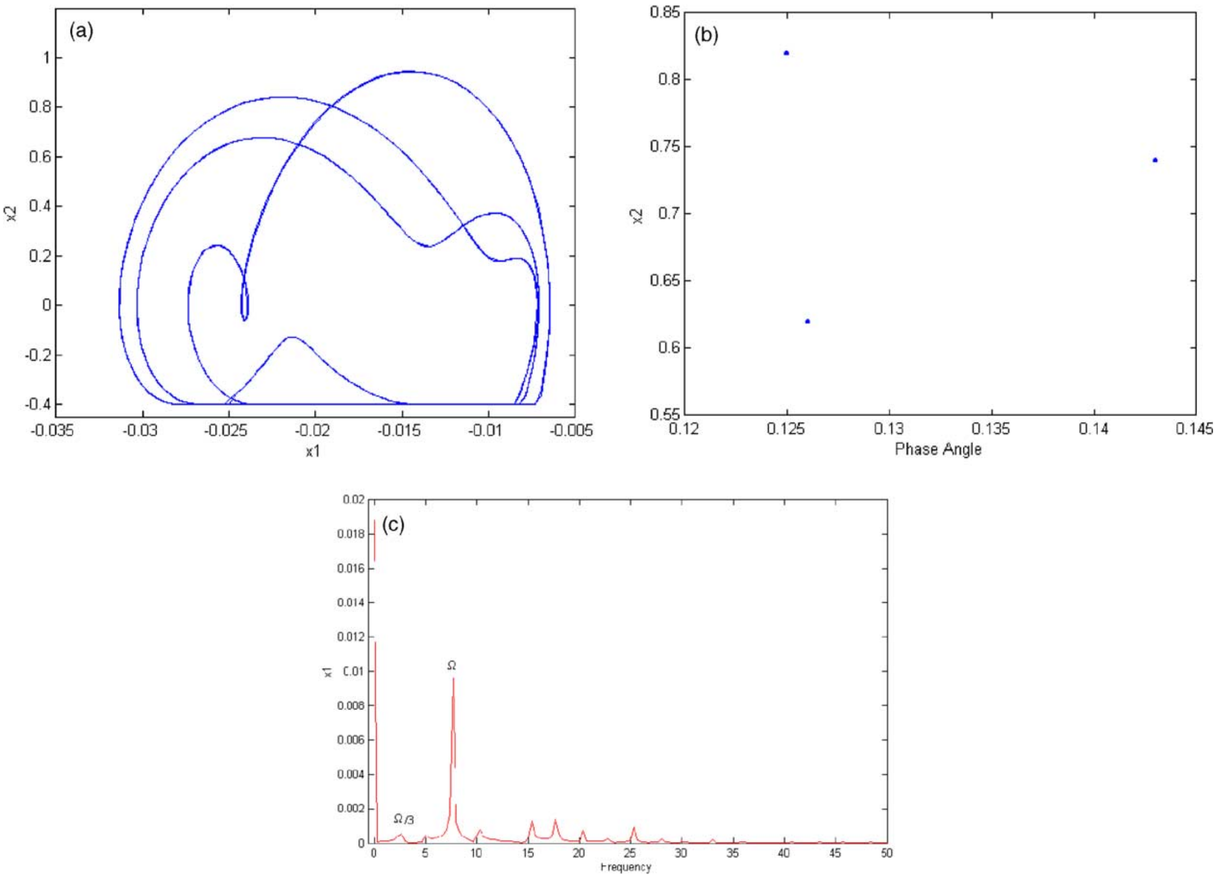

Fig. 9. Period-three orbit for $\dot{\psi}_{\mathrm{D}} = 0.398$ (rad/s): (a) phase portrait; (b) Poincare map; and (c) frequency spectrum.

We consider Eq. (7a) with state feedback control on the driver's side can be written as follows:

$$
\begin{aligned}
&\text{when} \\
&\quad \dot{z}_{\mathrm{D}} \neq 0, \\
&\quad \dot{x}_1 = x_2, \\
&\quad \dot{x}_2 = (-R_{\mathrm{D}} - D_{\mathrm{D}} - M_{\mathrm{D}}(\dot{z}_{\mathrm{D}}))/I_{\mathrm{D}} + Gx_2, \\
&\text{when} \\
&\quad \dot{z}_{\mathrm{D}} = 0, |R_{\mathrm{D}}| \geqslant N_{\mathrm{D}} l_{\mathrm{D}} \mu_0, \\
&\quad \dot{x}_1 = x_2, \\
&\quad \dot{x}_2 = (-R_{\mathrm{D}} - D_{\mathrm{D}} - M_{\mathrm{D}}(\dot{z}_{\mathrm{D}}))/I_{\mathrm{D}} + Gx_2, \\
&\text{when} \\
&\quad \dot{z}_{\mathrm{D}} = 0, |R_{\mathrm{D}}| < N_{\mathrm{D}} l_{\mathrm{D}} \mu_0, \\
&\quad x_2 = -\dot{\psi}_{\mathrm{D}}, \\
&\quad \dot{x}_1 = x_2, \\
&\quad \dot{x}_2 = 0 + Gx_2.
\end{aligned}
\tag{15a}
$$

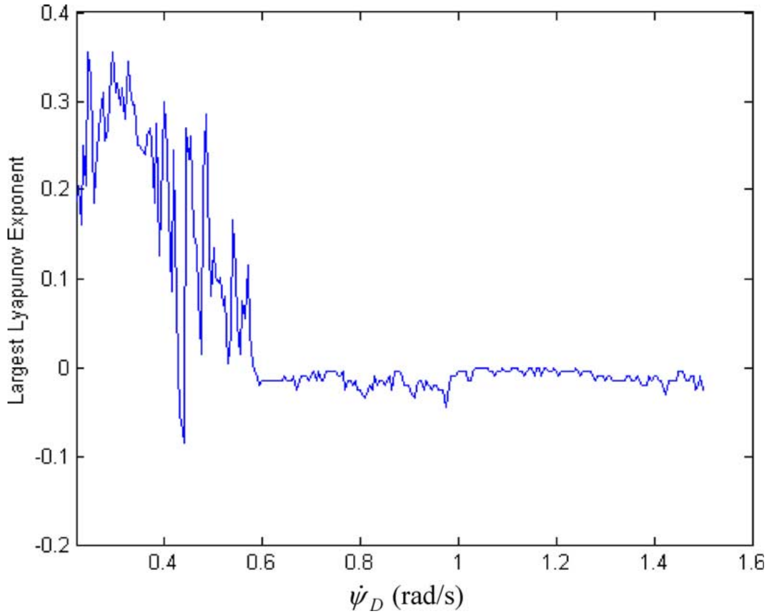

Fig. 10. The evolutions of the largest Lyapunov exponent.

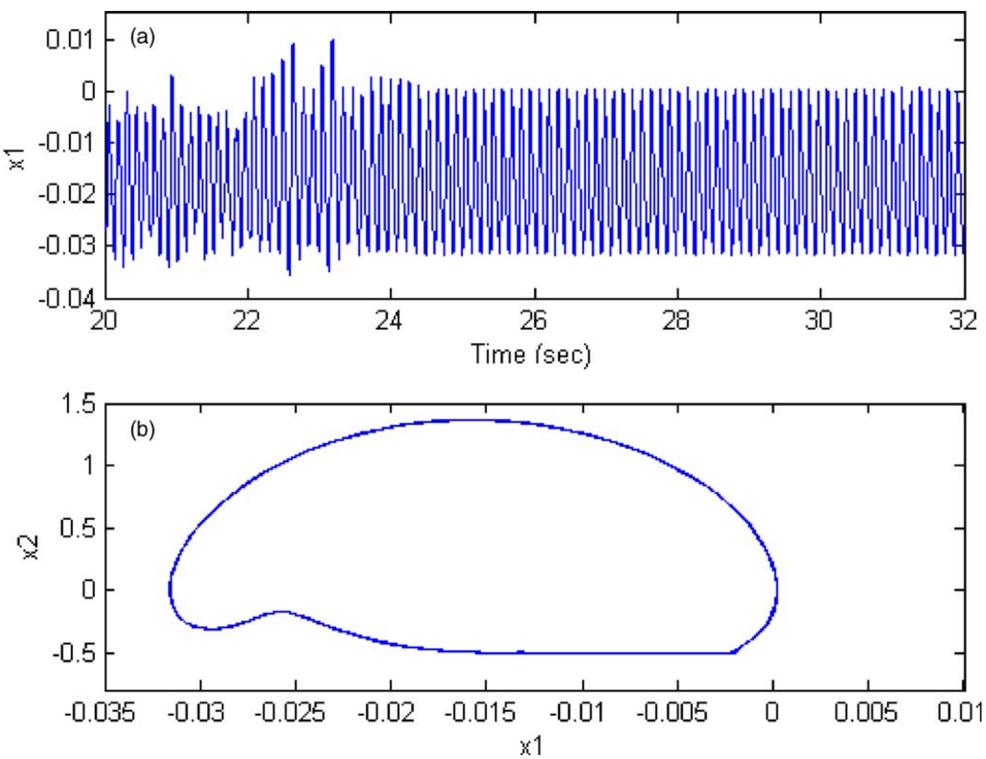

Fig. 11. Period-one motion of the wiper system with state feedback control ($G = 8.8$). The control signal is added after 22 s: (a) time response; and (b) controlled orbit.

We consider Eq. (7b) with state feedback control on the passenger's side can be written as follows:

$$\begin{aligned}
&\text{when}\\
&\quad \dot{z}_P \neq 0,\\
&\quad \dot{x}_3 = x_4,\\
&\quad \dot{x}_4 = (-R_P - D_P - M_P(\dot{z}_P))/I_P + Gx_4,\\
&\text{when}\\
&\quad \dot{z}_P = 0, |R_P| \geqslant N_P l_P \mu_0,\\
&\quad \dot{x}_3 = x_4,\\
&\quad \dot{x}_4 = (-R_P - D_P - M_P(\dot{z}_P))/I_P + Gx_4,\\
&\text{when}\\
&\quad \dot{z}_P = 0, |R_P| < N_P l_P \mu_0,\\
&\quad x_4 = -\dot{\psi}_P,\\
&\quad \dot{x}_3 = x_4,\\
&\quad \dot{x}_4 = 0 + Gx_4.
\end{aligned} \tag{15b}$$

When $G = 0$ and $\dot{\psi}_D = 0.5$, Eqs. (15) displays chaotic motion (see Fig. 7). In order to convert the dynamics of system (15) from chaotic motion to the periodic motion, the chosen feedback gain G is 8.8. The time response of x_1 is shown in Fig. 11(a) where the state feedback control is added after 22 s. Fig. 11(b) shows the phase portrait of the system after control.

6. Conclusions

Our main purpose in this paper is to study chaotic attitude behavior and the problem of chaos control on an automotive wiper system. Numerical methods including time responses, Poincare maps, frequency spectra and the largest Lyapunov exponent are employed to obtain the characteristics of the nonlinear wiper system. Many nonlinear and chaotic phenomena have been displayed in bifurcation diagrams. From this diagram, we can find that the chaotic motion appears a lot in lower wiping speed for wiper system. In order to examine whether the system is in chaotic motion or not, the Lyapunov exponent will be the most useful method to diagnostics for chaotic system. The method of estimation of the largest Lyapunov exponent for wiper system uses the properties of synchronization phenomenon. Finally, in order to effectively improve the performance of wiper system or avoid the chaotic motions, the state-feedback control method is applied to suppress chaotic motion.

Acknowledgement

This research was supported by the National Science Council in Taiwan, Republic of China, under project number NSC 91-2745-P-212-003.

References

Benettin, G., Galgani, L., Giorgilli, A., Strelcyn, J.M., 1980a. Lyapunov exponents for smooth dynamical systems and Hamiltonian systems; a method for computing all of them. Part I: Theory. Meccanica 15, 9–20.

Benettin, G., Galgani, L., Giorgilli, A., Strelcyn, J.M., 1980b. Lyapunov exponents for smooth dynamical systems and Hamiltonian systems; a method for computing all of them. Part II: Numerical application. Meccanica 15, 21–30.

Cai, C., Xu, Z., Xu, W., 2002. Converting chaos into periodic motion by state feedback control. Automatica 38, 1927–1933.

Dupont, P.E., 1991. Avoiding stick-slip in position and force control through feedback. In: Proceedings of the 1991 IEEE, International Conference on Robotics and Automation, Sacramento, California-April, pp. 1470–1475.

Feeny, B.F., Moon, F.C., 2000. Quenching stick-slip chaos with dither. Journal of Sound and Vibration 273 (1), 173–180.

Galvanetto, U., 2001. Flexible control of chaotic stick-slip mechanical systems. Computer Methods in Applied Mechanics and Engineering 190, 6075–6087.

Hinrichs, N., Oestreich, M., Popp, K., 1997. Dynamics of oscillators with impact and friction. Chaos, Solitons & Fractals 8 (4), 535–558.

Mokhtar, M.O.A., Younes, Y.K., Mahdy, T.H.E.L., Attia, N.A., 1998. A theoretical and experimental study on the dynamics of sliding bodies with dry conformal contacts. Wear 218, 172–178.

Muller, P., 1995. Calculation of Lyapunov exponents for dynamical systems with discontinuities. Chaos, Solitons & Fractals 5 (9), 1671–1681.

Oancea, V.G., Laursen, T.A., 1998. Investigations of low frequency stick-slip motion: experiments and numerical modeling. Journal of Sound and Vibration 213 (4), 577–600.

Shimada, I., Nagashima, T., 1979. A numerical approach to ergodic problems of dissipative dynamical systems. Progress of Theoretical Physics 61, 1605–1616.

Stefanski, A., 2000. Estimation of the largest Lyapunov exponent in systems with impact. Chaos, Solitons & Fractals 11 (15), 2443–2451.

Suzuki, R., Yasuda, K., 1995. Analysis of chatter vibration in an automotive wiper assembly (derivation of approximate solutions and study of vibration characteristics). Transactions of the Japan Society for Mechanical Engineering (in Japanese) 61 (588), C3203–3209.

Suzuki, R., Yasuda, K., 1998. Analysis of chatter vibration in an automotive wiper assembly. JSME International Journal, Series C 41 (3), 616–620.

Tarng, Y.S., Cheng, H.E., 1995. An investigation of stick-slip friction on the contouring accuracy of CNC machine tools. International Journal of Machine Tools and Manufacture 35, 565–576.

Wolf, A., Swift, J.B., Swinney, H.L., Vastano, J.A., 1985. Determining Lyapunov exponents from a time series. Physica D 16, 285–317.